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1998 J. Phys. A: Math. Gen. 31 5585

(http://iopscience.iop.org/0305-4470/31/25/009)

J. Phys. A: Math. Gen. **31** (1998) 5585–5598. Printed in the UK PII: S0305-4470(98)92144-8

Response of a quantum system to a time-dependent external field and dynamical symmetry of the system

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Received 2 March 1998

Abstract. The response of a quantum system to a time-dependent periodic external field is investigated in connection with the dynamical symmetry breaking and level dynamics of the adiabatic states of the system. The main results are as follows. (A) When the periodic external field preserves the dynamical symmetry of the system, its response is like that of elastic matter. (B) When the periodic external field breaks the dynamical symmetry, several cases may occur: (a) in the adiabatic limit, the system still responds elastically; (b) if the initial state is an eigenstate of the evolution operator $U(T)$ for one period T of the external field, the system evolves in time cyclically and responds quasi-elastically; (c) if the initial state is not an eigenstate of the evolution operator $U(T)$, the system evolves in time non-cyclically and responds non-elastically. The detailed non-elastic behaviour depends on the statistical nature of the adiabatic eigenstates of the system. If the adiabatic spectrum is chaotic, the non-elastic response is expected to be strongly dissipative. The avoided level crossings of the adiabatic eigenstates play a crucial role in both producing chaoticity of the adiabatic levels and causing dissipation of the non-elastic response. The non-elastic role played by the adiabatic progressive phase is also addressed. Computer experiments are performed for the su(2) dynamical model to illustrate the above general results.

1. Introduction

The response of a quantum system to a time-dependent external field is of general interest in many branches of physics, such as solid state physics, atomic and molecular physics, and nuclear physics. A large variety of interesting phenomena have been found in this context. Generally, as a time-dependent external field acts on a quantum system, its response may fall into one of several categories such as elastic, viscous or elastoplastic [1]. In nuclear physics, as two heavy ions are colliding, a time-dependent mean field results. The dissipative behaviour of the two nuclei depends on both the time-dependent mean field and the two-body residual interactions [2]. Intuitively, both the time-dependent mean field and the residual interactions will destroy the good quantum numbers of individual nucleons and may drive them to chaotic motion. As the nucleonic motion changes from regular to irregular modes, energy and angular momentum of the collective motion of the nucleus will dissipate into single-particle motion. In this paper, we shall use group theoretical methods to study the mechanism that governs the above diverse phenomena by an investigation of a quantum system which possesses a dynamical group, i.e. whose Hamiltonian is a function of certain

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Lie group generators. In what follows, we shall show that the response of a quantum system is closely related to the preserving or breaking of the dynamical symmetry of the system due to the external field. For simplicity, we assume that the time-dependent external field is periodic.

2. General formalism

Consider a quantum system whose dynamics is dictated by a time- periodic Hamiltonian,

$$H(t) = H(\alpha(t)) \tag{1}$$

where α is a periodic function of t with frequency ω. The time dependence of the Hamiltonian can be induced by a time-periodic external field,

$$H(t) = H_0 + V(\alpha(t)). \tag{2}$$

Suppose the dynamical group of the unperturbed Hamiltonian H_0 is G with $\{X_\nu\}$ as its generators and $\{C, C_i\}$ as a complete set of commuting operators, usually chosen according to some group chain [3, 4]. The dynamical symmetry [5, 6] G of H_0 means that H_0 is a function of $\{C, C_i\}$,

$$H_0 = H_0(X_\nu) = H_0(C, C_i) \tag{3}$$

where C and C_i are the Casimir operators of some group chain

$$G(C) \supset G_i(C_i) \tag{4}$$

with the property

$$[C, C_i] = 0 \qquad [C_i, C_j] = 0. \tag{5}$$

If G is also the dynamical group of $H(t)$, then $H(t)$ can be represented as function of the group generators X_ν,

$$H(t) = H(X_\nu, \alpha(t)) = H_0(C, C_i) + V(X_\nu, \alpha(t)). \tag{6}$$

The time-periodic perturbation $V(t)$ may either preserve or destroy the dynamical symmetry of the system. Assume ϕ_n and $n = \{n_1, n_2, \ldots, n_l\}$ are eigenstates and eigenvalues of $\{C, C_i\}$. From equation (3) it is evident that ϕ_n are also eigenstates of H_0, namely

$$H_0\phi_n = \epsilon_n(0)\phi_n. \tag{7}$$

For the time evolution of the system by the time-dependent Schrödinger equation,

$$\mathrm{i}\partial\psi(t)/\partial t = H(t)\psi(t) \tag{8}$$

we consider two cases: (A) $V(t)$ preserves the dynamical symmetry of H_0, i.e. $V(t)$ preserves the quantum numbers $\{n_1, n_2, \ldots, n_l\}$; (B) $V(t)$ breaks the dynamical symmetry of H_0, i.e. it destroys the quantum numbers $\{n_1, n_2, \ldots, n_l\}$.

(A) V(t) preserves the dynamical symmetry

We first prove a theorem which underlies the discussion of this section.

(1) *Theorem.* The necessary and sufficient condition for $V(t)$ to preserve the dynamical symmetry of H_0 is that $H(t)$ assumes the following form,

$$H(t) = H(C, C_i, \alpha(t)) = H_0(C, C_i) + V(C, C_i, \alpha(t)) \tag{9}$$

namely $V(t)$ is also a function of $\{C, C_i)\}$ only.

Proof. If $H = H(C, C_i, \alpha(t))$, then from equation (5) we have

$$[H(t), C] = 0 \qquad [H(t), C_i] = 0 \tag{10}$$

which means that $\{C, C_i\}$ are conserved operators and their quantum numbers $\{n_1, n_2, ..n_l\}$ are constant during the time evolution. Conversely, if $\{C, C_i\}$ are conserved under $H(t)$, then $\{C, C_i\}$ and $H(t)$ commute,

$$[H(t), C] = 0 \qquad [H(t), C_i] = 0 \tag{11}$$

hence $H(t)$ must be a function of $\{C, C_i\}$.

(2) *Dynamical behaviour of the system*

(i) Adiabatic eigensolutions ϕ_n of $H(t)$,

$$H(C, C_i, \alpha(t))\phi_n = \epsilon_n(\alpha)\phi_n \tag{12}$$

are also eigenstates of $\{C, C_i\}$, they form an irreducible representation basis of the group G,

$$\{C, C_i\}\phi_n = \{n_i\}\phi_n. \tag{13}$$

Thus ϕ_n are algebraic-geometric objects of the group G and independent of the parameter α,

$$\frac{\partial \phi_n}{\partial \alpha} = 0 \tag{14}$$

while the energies $\epsilon_n(\alpha)$ are dynamical quantities depending on α.

(ii) Time-dependent behaviour. Expand the general solution of the time- dependent Schrödinger equation in terms of ϕ_n,

$$\psi(t) = \sum_n b_n(t)\phi_n. \tag{15}$$

Inserting equation (15) into equation (8), one has together with equation (12)

$$\mathrm{i}\frac{\mathrm{d}b_n}{\mathrm{d}t} = b_n\epsilon_n(\alpha(t)) \tag{16}$$

and

$$b_n(t) = b_n(0)\exp\left[-\mathrm{i}\int_0^t \epsilon_n(\tau)\,\mathrm{d}\tau\right]. \tag{17}$$

The energy of the system is

$$E(t) = \langle\psi(t)|H(t)|\psi(t)\rangle = \sum_n |b_n(0)|^2\epsilon_n(\alpha(t)) = E(\alpha(t)) \tag{18}$$

which means that the system responds elastically to the external field $V(t)$. Furthermore, if $\psi(0) = \phi_n$, we have a stationary solution,

$$\psi(t) = \phi_n \exp\left(-\mathrm{i}\int_0^t \epsilon_n(\tau)\,\mathrm{d}\tau\right) = \phi_n \exp\left(-\mathrm{i}\int_0^t \langle\psi(\tau)|H(\tau)|\psi(\tau)\rangle\,\mathrm{d}\tau\right) \tag{19}$$

which means that the system possesses only a dynamical phase, but no Berry phase [7].

(B) V(t) breaks the dynamical symmetry

Then, according to the above theorem, $V(t)$ is not a function of $\{C, C_i\}$ but a function of $\{X_v\}$,

$$V(t) = V(X_v, \alpha(t)) \neq V(C, C_i, \alpha(t)). \tag{20}$$

Let $V(t)$ be periodic with the period $T = 2\pi/\omega$, $V(t+T) = V(t)$. Without loss of generality, we assume

$$V(T) = V(0) = 0. \tag{21}$$

The eigensolutions of H_0 and $H(\alpha(t))$ are

$$H_0\phi_n = \epsilon_n(0)\phi_n \qquad \langle\phi_n|\phi_m\rangle = \delta_{nm} \tag{22}$$

and

$$H(\alpha)\xi_p(\alpha) = E_p(\alpha)\xi_p(\alpha) \qquad \langle\xi_p|\xi_q\rangle = \delta_{pq}. \tag{23}$$

Equation (8) has the formal solution

$$\psi(t) = U(t)\psi(0) \tag{24}$$

where the time evolution operator is defined as usual,

$$U(t) = \hat{T}\exp\left(-\mathrm{i}\int_0^t H(\tau)\,\mathrm{d}\tau\right). \tag{25}$$

Consider the time evolution operator $U(T)$ for one period T of the external field $V(t)$. Its eigenvalue equation is

$$U(T)\eta_k(T) = \exp(-\mathrm{i}\theta_k(T))\eta_k(T) \tag{26}$$

where $\theta_k(T)$ is Floquet's index, and the n-period solution reads

$$\psi(nT) = [U(T)]^n\psi(0). \tag{27}$$

The dynamical evolution can be divided into two major categories: adiabatic evolution and non-adiabatic evolution.

(1) *Adiabatic evolution.* If one expands

$$\psi(t) = \sum_p a_p(t)\xi_p(\alpha) \tag{28}$$

the equation of motion for a_p is

$$\mathrm{i}\frac{\mathrm{d}a_p}{\mathrm{d}t} = E_p(\alpha(t))a_p(t) - \mathrm{i}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sum_q\left\langle\xi_p\left|\frac{\partial}{\partial\alpha}\right|\xi_q\right\rangle a_q(t). \tag{29}$$

In the adiabatic limit when $\frac{\mathrm{d}\alpha}{\mathrm{d}t} \to 0$, the off-diagonal part in equation (29) can be neglected so that

$$\mathrm{i}\frac{\mathrm{d}a_p}{\mathrm{d}t} = E_p(t)a_p(t) - \left\langle\xi_p(t)\left|\mathrm{i}\frac{\partial}{\partial t}\right|\xi_p(t)\right\rangle a_p(t) \tag{30}$$

which has the adiabatic solution

$$a_p(t) = \exp(-\mathrm{i}\theta_p(t))a_p(0) \qquad \theta_p(t) = \int_0^t\left(E_p(\tau) - \left\langle\xi_p\left|\mathrm{i}\frac{\partial}{\partial\tau}\right|\xi_p\right\rangle\right)\mathrm{d}\tau. \tag{31}$$

The energy is

$$E(t) = \langle\psi(t)|H(t)|\psi(t)\rangle = \sum_p |a_p(0)|^2 E_p(\alpha(t)) = E(\alpha(t)) \tag{32}$$

which indicates that in adiabatic evolution, the system responds elastically as in case (A), equation (18). The adiabatic evolution is of course related to the adiabatic Berry phase [7].

(2) *Non-adiabatic evolution [8].*

(i) Cyclic (or recurrent) evolution requires the initial state to be an eigenstate of $U(T)$,

$$\psi(0) = \eta_k. \tag{33}$$

This leads, according to equations (26), (27), to the recurrent solution

$$\psi(nT) = \exp(-\mathrm{i}n\theta_k(T))\eta_k. \tag{34}$$

The cyclic evolution is related to the non-adiabatic Berry phase [8]. For one period the dynamical phase is

$$\theta_k^d(T) = \int_0^T \langle\psi_k(t)|H(t)|\psi_k(t)\rangle \,\mathrm{d}t \tag{35}$$

and the Berry phase is

$$\theta_k^B(T) = -(\theta_k(T) - \theta_k^d(T)). \tag{36}$$

The energy is, by virtue of equations (21), (34),

$$E(nT) = \langle\psi(nT)|H(nT)|\psi(nT)\rangle = \langle\eta_k|H_0|\eta_k\rangle = E(0). \tag{37}$$

This indicates that in the cyclic evolution, the energy of the system changes in time within the period T and at each cycle the energy returns to its initial value. Since $E(t)$ is in general not a function of α, the response of the system is not really elastic. However, in view of the energy restoration after each period, one can refer to the system's response as quasi-elastic.

(ii) For non-cyclic evolution, the initial state is not an eigenstate of $U(T)$,

$$\psi(0) \neq \eta_k \tag{38}$$

which leads to a non-cyclic (non-recurrent) solution. Let

$$\psi(0) = \sum_k C_k^{(0)}\eta_k \tag{39}$$

then

$$\psi(nT) = \sum_m D_{0m}^n \phi_m \tag{40}$$

where from equations (26), (27)

$$D_{0m}^n = \sum_k C_k^{(0)} d_{km} \exp(-\mathrm{i}n\theta_k) \qquad \eta_k = \sum_m d_{km}\phi_m. \tag{41}$$

The energy of the system is

$$E(nT) = \langle\psi(nT)|H(nT)|\psi(nT)\rangle = \sum_m P_{0m}(nT)\epsilon_m(0) \tag{42}$$

where the distribution probability P_{0m} is

$$P_{0m}(nT) = |D_{0m}^n|^2. \tag{43}$$

If the system is initially in the ground state of H_0, namely

$$\psi(0) = \phi_0 \qquad \text{with energy } \epsilon_0(0), \tag{44}$$

then the system will absorb energy from the external field because

$$E(nT) \geqslant \epsilon_0(0). \tag{45}$$

To study the non-elastic behaviour of the system, one has to make a clear-cut separation of the non-elastic effect from the elastic one. To this end we use the instantaneous adiabatic eigenstates ξ_n as a working basis. Let us expand the time-dependent solution $\psi(t)$ in terms of this basis,

$$\psi_{n_0}(t) = \sum_n A_{nn_0}(t) \exp(-\mathrm{i}\theta_n(t))\xi_n(\alpha(t)) \tag{46}$$

where the subscript n_0 refers to the quantum number of the initial state, and θ_n is the progressive phase of the adiabatic eigenstate which is given by equation (31).

Inserting equation (46) into equation (8), we obtain the equation of motion for A_{nn_0}

$$\frac{\mathrm{d}A_{nn_0}}{\mathrm{d}t} = \sum_{m\neq n} K_{nm} A_{mn_0}. \tag{47}$$

For $V(\alpha) = \alpha V$, equation (47) is characterized by the kernel

$$K_{nm} = -\frac{\mathrm{d}\alpha}{\mathrm{d}t} \frac{\langle \xi_m(\alpha)|V|\xi_n(\alpha)\rangle^*}{E_m(\alpha) - E_n(\alpha)} \exp[-\mathrm{i}(\theta_m(t) - \theta_n(t))] \tag{48}$$

which determines the transition rate from the progressive adiabatic state $\exp(-\mathrm{i}\theta_m)\xi_m$ to $\exp(-\mathrm{i}\theta_n)\xi_n$ caused by the dynamical symmetry-breaking interaction V. From equation (48) we know that the transition kernel K_{nm} is completely determined by the adiabatic eigenstates, their progressive phases, and the rate of change of the parameter α. This means that the non-elastic behaviour of the system is related to its adiabatic eigenstates. We call this kind of relation a dynamical correlation between adiabatic and non-adiabatic processes.

The energy of the system is

$$E^{(n_0)}(t) = \langle \psi_{n_0}(t)|H(\alpha(t))|\psi_{n_0}(t)\rangle = \sum_n |A_{nn_0}(\alpha(t), t)|^2 E_n(\alpha(t)). \tag{49}$$

Since $E_n(\alpha(t))$ depends on time only through α while the A_{nn_0} have a more complicated time dependence, equation (49) shows a clear-cut separation of elastic and non-elastic effects. If quantum transitions are completely neglected, i.e. $K_{nm} = 0$, then $A_{nn_0}(t) = A_{nn_0}(0)$ and

$$E^{(n_0)}(t) = \sum_n |A_{nn_0}(0)|^2 E_n(\alpha(t)) = E^{(n_0)}(\alpha(t)). \tag{50}$$

Equation (50) is precisely the expression of the elastic energy. As quantum transitions are included, equation (47) is not integrable in α-space, and $E(t)$ is not a function of $\alpha(t)$ only, since $\theta_n(t)$ is a function of both α and t . Therefore

$$E(t) = E(\alpha(t), t) \neq E(\alpha(t)) \tag{51}$$

the system responds non-elastically. The above analysis tells us that it is the progressive phase which is not integrable in α-space, making the system respond non-elastically.

The dynamical adiabatic and non-adiabatic correlation can be exploited further, if a comparison between the equations of motion for A_{nn_0}, equation (47), and the level dynamical equations [9] for the adiabatic eigenstates, equation (30), is made as follows: we have assumed that H_0 is integrable and has a complete set of good quantum numbers and that V breaks the dynamical symmetry of H_0. Thus $H(\alpha(t))$ is non- integrable and the good quantum numbers of H_0 are destroyed by V. To study the non-integrable system with Hamiltonian $H(\alpha(t))$, we employ the unperturbed eigenstates ϕ_n as a working basis. Expanding $\xi_n(\alpha)$ in terms of ϕ_n,

$$\xi_n(\alpha) = \sum_r B_{nr}(\alpha)\phi_r \tag{52}$$

we obtain from the eigenvalue equation of $H(\alpha)$, equation (23), the dynamical equation for B_{nr},

$$\frac{\mathrm{d}B_{nr}(\alpha)}{\mathrm{d}\alpha} = -\sum_{m\neq n} \frac{\langle \xi_m | V | \xi_n \rangle}{E_m - E_n} B_{mr}. \tag{53}$$

For numerical calculation, ξ_m in equation (53) is expanded in terms of ϕ_r as shown in equation (52) so that equation (53) becomes explicitly nonlinear. With $\alpha = \alpha(t)$, equation (53) can be written as

$$\frac{\mathrm{d}B_{nr}}{\mathrm{d}t} = \sum_{m\neq n} \kappa_{nm} B_{mr} \tag{54}$$

where the kernel κ_{nm} reads

$$\kappa_{nm} = -\frac{\mathrm{d}\alpha}{\mathrm{d}t} \frac{\langle \xi_m | V | \xi_n \rangle}{E_m - E_n}. \tag{55}$$

Equation (54) with kernel equation (55) is the basic equation of level dynamics [9]. Comparing equations (47), (48) with equations (54), (55), we find that the structure of the two equations is similar. The only difference is at the kernels: for the general time-dependent case, the interaction matrices are calculated by using the progressive adiabatic states $\exp(-\mathrm{i}\theta_n(t))\xi_n(\alpha(t))$, while the progressive phase $\exp(-\mathrm{i}\theta_n(t))$ is not involved for the adiabatic case. Because of the absence of the progressive phase in κ_{nm}, equation (54) is integrable in α-space. In contrast, the presence of the progressive phase in K_{nm} makes equation (47) non-integrable with respect to α. It is the progressive phase, which is not integrable in α-space, that makes the system respond non-elastically. In short, the elasticity is related to the integrability of the dynamical equation in α-space, while the non-elasticity is due to the non-integrability of the dynamical equation in α-space.

In level dynamics [9] it is shown that if a system, starting from an integrable Hamiltonian H_0, undergoes an adiabatic symmetry-breaking evolution, the properties of its spectrum strongly depend on the avoided crossings of the adiabatic levels. During the adiabatic evolution, if the system has developed only a few avoided level crossings, the kernel κ_{nm} contains mainly long-range interactions. By summation, it produces a mean field which yields a smooth deformation of the adiabatic level spectrum. After unfolding [10], the mean field effect is removed, and the level spectrum shows a regular behaviour. In contrast, during the course of adiabatic evolution as the system has developed many avoided level crossings, the kernel κ_{nm} will comprise a large number of short-range collisions which in turn produce enormous local fluctuations in the adiabatic level spectrum and make the spectrum chaotic with Gaussian orthogonal ensemble (GOE) statistics [10].

Since the kernels K_{nm} and κ_{nm} have the same adiabatic energy denominators, the avoided level crossings play the same role in producing the fluctuations of the solution of the dynamical equations for both the adiabatic and non-adiabatic cases. From the dynamical adiabatic and non- adiabatic correlation, we expect that the regular system and the chaotic system will give different responses to the time-dependent external perturbation: the response of a regular system will be elastic, while the response of a chaotic system will show non-elastic (dissipative) behaviour.

3. Computer experiments

In this section, we shall present some numerical results from computer experiments to illustrate the above general conclusions.

We employ the su(2) dynamical model which was used previously to study the transition from regular to irregular quantum motion by means of level dynamics [9]. The Hamiltonian of the model reads

$$H(t) = H_0 + \alpha(V^{\mathrm{dia}} + V^{\mathrm{off}}) \tag{56}$$

where the regular (integrable) part H_0 is a function of the su(2) Cartan operator J_0,

$$H_0 = 2j \sin\left[\frac{1}{3}\arcsin\left(\frac{J_0}{j}\right)\right]. \tag{57}$$

It has a dynamical symmetry and good quantum numbers j and m. The perturbation which destroys the dynamical symmetry and the good quantum number m, can be divided into two parts: the diagonal part V^{dia} commutes with H_0 and keeps its dynamical symmetry as well as the related good quantum number m; the off-diagonal part V^{off} breaks the dynamical symmetry and destroys the good quantum number m. They read

$$V^{\mathrm{dia}} = (-)^{J_0}\left[\beta_1 \cos\left(2\pi k_1 \frac{J_0}{j}\right) + \beta_2 \cos\left(2\pi k_2 \frac{J_0}{j}\right)\right] \tag{58}$$

$$V^{\mathrm{off}} = (J_+ + J_-)/j. \tag{59}$$

As shown in [9], when the parameter set assumes the following values,

$$\beta_1 = 0.5 \qquad \beta_2 = -1.0 \qquad k_1/j = \tfrac{8}{500} \qquad k_2/j = \tfrac{28}{500} \qquad \alpha = 2.3 \tag{60}$$

the system becomes chaotic with GOE statistics [10], while for $\alpha = 0$ it is evident that the system is integrable and regular.

Now we consider the time-dependent case. Let

$$\alpha = \alpha(t) = \alpha_0 + \Delta\alpha \sin\omega t \tag{61}$$

with the last term due to a time-periodic external perturbation exerted on the system.

The time-dependent Schrödinger equation (8) can be solved in the regular basis $|m\rangle$ of the eigenstates of J_0 . Expand the state $|\psi_{m_0}(t)\rangle$ evolving from the initial state $|m_0\rangle$ in terms of $|m\rangle$,

$$|\psi_{m_0}(t)\rangle = \sum_m C_{m_0 m} \exp(-\mathrm{i}\theta_m(t))|m\rangle \tag{62}$$

where $\theta_m(t)$ of equation (31) reduces to

$$\theta_m(t) = \int_0^t \epsilon_m(\alpha(\tau))\,\mathrm{d}\tau. \tag{63}$$

Here $\epsilon_m(\alpha(t))$ and $|m\rangle$ are the solutions of the following equation:

$$(H_0 + \alpha(t)V^{\mathrm{dia}})|m\rangle = \epsilon_m(\alpha(t))|m\rangle \tag{64}$$

with

$$\epsilon_m(\alpha(t)) = \langle m|H_0 + \alpha(t)V^{\mathrm{dia}}|m\rangle = 2j \sin\left[\frac{1}{3}\arcsin\left(\frac{m}{j}\right)\right]$$
$$+\alpha(t)(-1)^m\left[\beta_1 \cos\left(2\pi\frac{k_1 m}{j}\right) + \beta_2 \cos\left(2\pi\frac{k_2 m}{j}\right)\right]. \tag{65}$$

From equations (8) and (62) we obtain the equation of motion for $C_{m_0 m}(t)$,

$$\frac{\mathrm{d}C_{m_0 m}(t)}{\mathrm{d}t} = -\mathrm{i}\sum_n C_{m_0 n}(t)\alpha(t)\langle m|V^{\mathrm{off}}|n\rangle \exp[-\mathrm{i}(\theta_n(t) - \theta_m(t))] \tag{66}$$

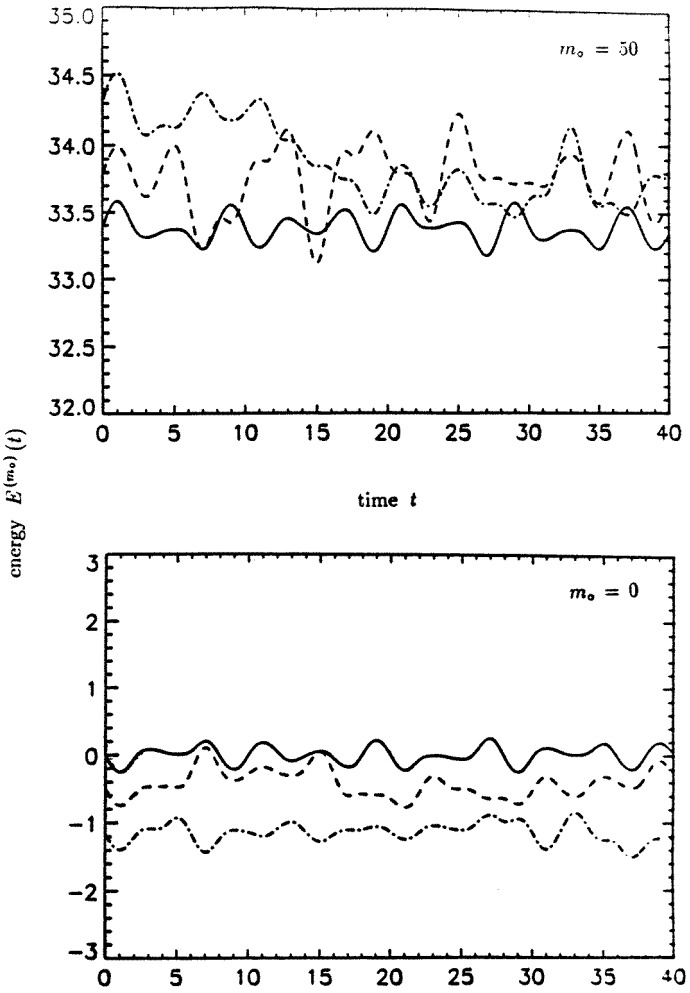

Figure 1. Time dependence of the energy $E^{(m_0)}(t)$ for $\omega = \pi/2$ with $m_0 = 0$ (lower part) and $m_0 = 50$ (upper part). Full curve: $\alpha_0 = 0$; broken curve: $\alpha_0 = 1.0$; chain curve: $\alpha_0 = 2.3$.

which is an analogue of equation (47) but in the unperturbed basis rather than the adiabatic basis.

To study the effect of the external perturbation on both regular and chaotic systems, the following quantities are used:

(i) the energy of $\psi_{m_0}(t)$ defined as

$$E^{(m_0)}(t) = \langle\psi_{m_0}(t)|H(\alpha(t))|\psi_{m_0}(t)\rangle = \sum_n |C_{m_0 n}|^2 \epsilon_n(\alpha(t)) + \sum_{n\neq m} C^*_{m_0 n}(t) C_{m_0 m}(t)\alpha(t)\langle n|V^{\text{off}}|m\rangle \exp[-\mathrm{i}(\theta_m(t) - \theta_n(t))] \tag{67}$$

(ii) the variance of angular momentum defined as usual,

$$\sigma^2_{m_0}(t) = \langle\psi_{m_0}(t)|(J_0 - \langle\psi_{m_0}(t)|J_0|\psi_{m_0}(t)\rangle)^2|\psi_{m_0}(t)\rangle$$

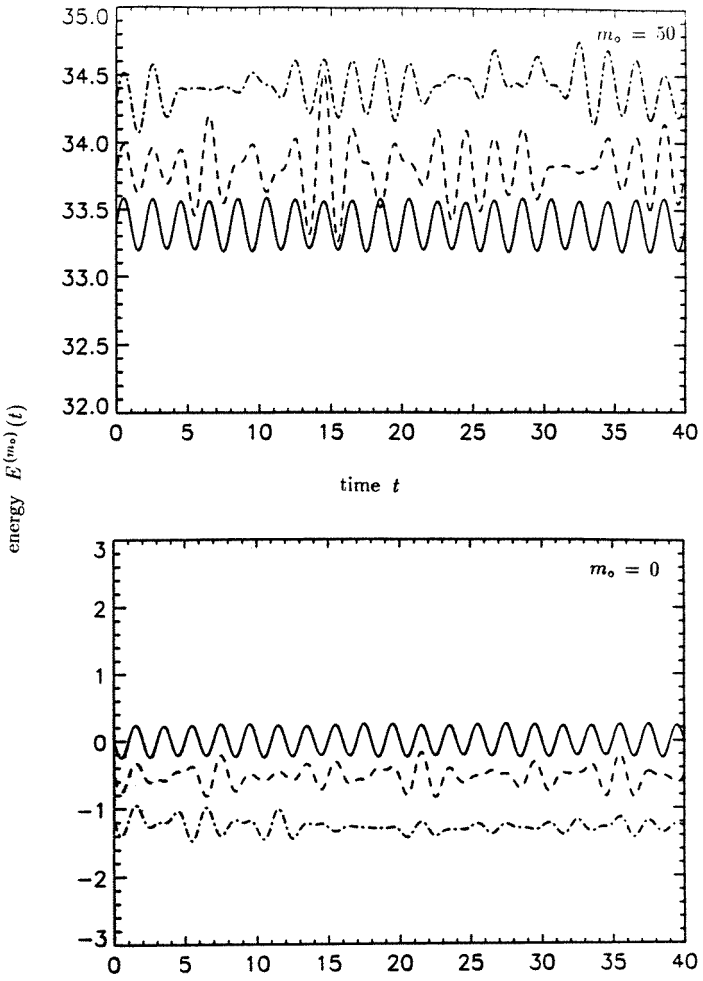

Figure 2. Time dependence of the energy $E^{(m_0)}(t)$ for $\omega = \pi$ with $m_0 = 0$ (lower part) and $m_0 = 50$ (upper part). Full curve: $\alpha_0 = 0$; broken curve: $\alpha_0 = 1.0$; chain curve: $\alpha_0 = 2.3$.

$$= \sum_m |C_{m_0 m}(t)|^2 m^2 - \left(\sum_m |C_{m_0 m}(t)|^2 m \right)^2 \tag{68}$$

(iii) the time evolution of the distribution of N fermions over the eigenstates $|m\rangle$ of the operator $(H_0 + \alpha(t)V^{\text{dia}})$, equation (64), under the perturbation $\alpha(t)V^{\text{off}}$, equation (59),

$$P_m^{(m_0)}(t) = |C_{m_0 m}(t)|^2 \qquad \text{with } \sum_m |C_{m_0 m}(t)|^2 = N, \text{ for } m \geqslant -j. \tag{69}$$

Before we can study the effect of the external, time-periodic perturbation, we have to specify appropriate values of the external perturbation strength $\Delta\alpha$ and of the frequency ω. Since we look for distinctive responses of the regular ($\alpha_0 = 0$) and chaotic ($\alpha_0 = 2.3$) systems, $\Delta\alpha$ should be large enough to produce a strong effect on the system. On the other hand, $\Delta\alpha$ must be small enough in order to avoid a situation where regular and chaotic regimes overlap. To meet these two conditions we assume $\Delta\alpha = 0.5$ so that, in view of the above α_0-values, $\alpha(t) = \alpha_0 + \Delta\alpha \sin \omega t$ keeps the distinction between regular and chaotic cases.

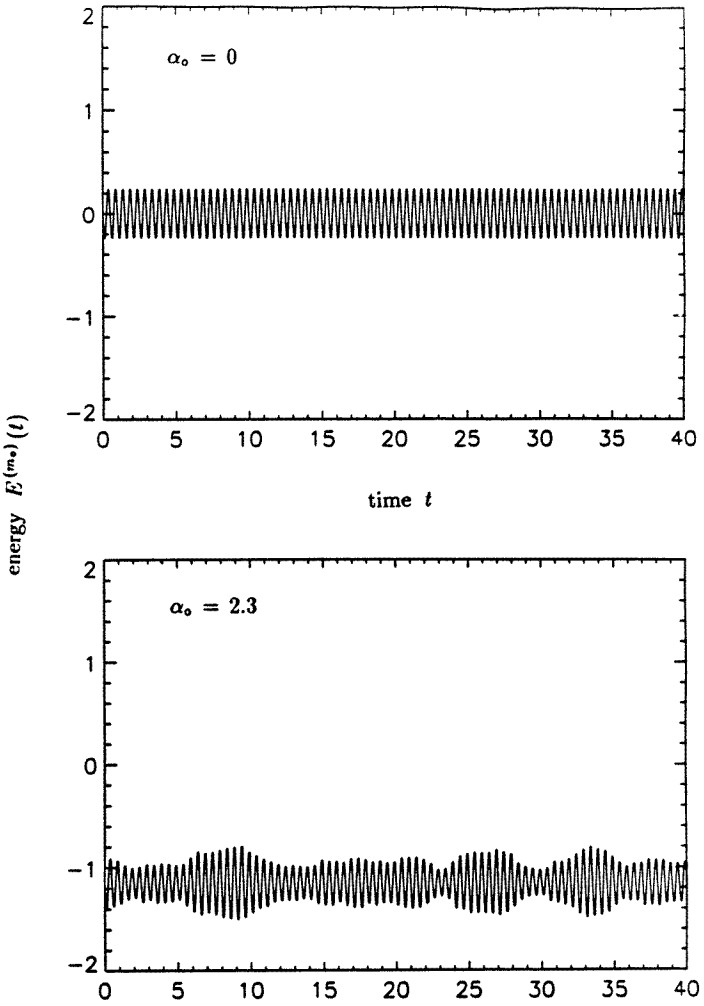

Figure 3. Time dependence of the energy $E^{(m_0)}(t)$ for $\omega = 4\pi$ and $m_0 = 0$. Top: $\alpha_0 = 0$; bottom: $\alpha_0 = 2.3$.

To make an appropriate choice of ω, we have to consider the intrinsic frequencies of the system. From [9] we know that the average level spacing of the system under study is $D \approx 1$ with a range from 0 to 2.5. Hence the corresponding intrinsic frequencies are in some interval $[0, 2.5]$ which constitutes a resonance window for the external time-periodic perturbation. If we choose ω in this window, say $\omega \leqslant \pi/2$, both the regular and the chaotic systems will respond resonantly, and their distinction fades. Figure 1 presents such a resonant case for $\omega = \pi/2$ where the regular and chaotic cases show similar structures for the energy $E^{(m_0)}(t)$. In the following we present numerical results for $j = 500$ obtained with $\Delta\alpha = 0.5$ and $\omega = \pi, 4\pi$.

Figure 2 shows the energy $E^{(m_0)}(t)$ for $m_0 = 0$ and 50 as quantum numbers of the initial state $|m_0\rangle$. While for the regular case ($\alpha_0 = 0$) the energy follows closely the time dependence of $\alpha(t)$, its behaviour becomes increasingly irregular as one approaches the chaotic regime ($\alpha_0 = 2.3$), and the non-elastic energy deviation increases. Further

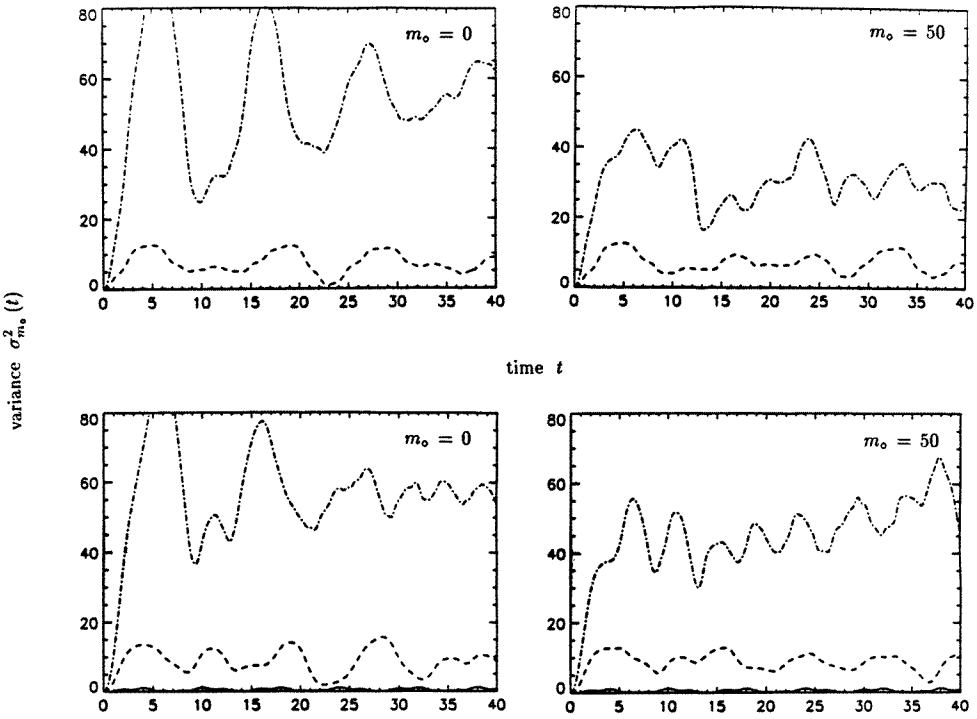

Figure 4. Variance $\sigma^2_{m_0}(t)$ of angular momentum for $m_0 = 0$ and 50 with $\omega = \pi/2$ (lower part) and $\omega = \pi$ (upper part). Full curve: $\alpha_0 = 0$; broken curve: $\alpha_0 = 1.0$; chain curve: $\alpha_0 = 2.3$.

increase of ω (figure 3) results in a low-frequency modulation of the $\sin \omega t$ behaviour of the external perturbation due to the relatively low intrinsic frequencies of the system. Hence the average of $E^{(m_0)}(t)$ over one period of the external perturbation no longer exhibits the difference between the regular and chaotic cases. The angular momentum dispersion (figure 4) $\sigma_{m_0}(t)$ remains almost zero during time evolution for the regular case. For the chaotic case, $\sigma_{m_0}(t)$ increases steeply with time t and tends to saturate at some non-zero value after several pronounced oscillations. Finally we consider a system of independent fermions which initially occupy the 50 lowest orbits $|m\rangle$ with energies ϵ_m for $j = 500$. These orbits are mixed during time evolution by the non-diagonal coupling term of $H(t)$. The occupation probability $P_m^{(m_0)}(t)$ of these orbits clearly distinguishes between the regular and chaotic case (figure 5); while for $\alpha_0 = 1.0$ the initial (rectangular) Fermi distribution is only slightly smeared out, the effect is much more dramatic for the chaotic case with $\alpha_0 = 2.3$.

4. Summary

The response of a quantum system to a time-dependent, external perturbation has been studied systematically. As one would expect, the system responds elastically, if the perturbation preserves the symmetry of the unperturbed system. This is also true for a symmetry-breaking perturbation in the adiabatic limit. In those two cases the energy strictly follows the time-dependence $\alpha(t)$ of the perturbation, $E(t) = E(\alpha(t))$, the system responds *elastically*. For non-adiabatic evolution the system will in general react *non-elastically*.

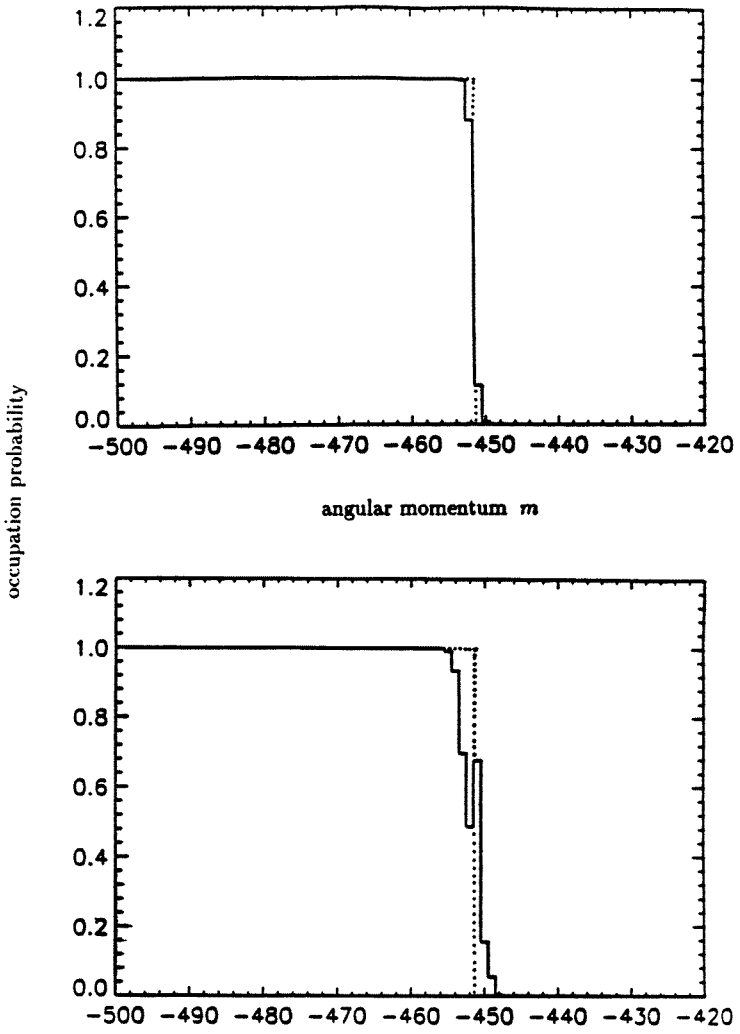

Figure 5. Occupation probability $P_m^{(m_0)}(t)$ of orbits $|m\rangle$, for 50 fermions in the same spin state, for the regular ($\alpha_0 = 1.0$) and chaotic ($\alpha_0 = 2.3$) cases at time $t = 104$. The dotted curve indicates the initial (rectangular) distribution.

The detailed behaviour depends on the spectrum of the adiabatic eigenstates of the total system: if the spectrum is regular, the response is almost elastic, while for chaotic spectrum the response is inelastic and dissipative. It is this property where the present study of the response to a time-dependent perturbation connects to the problem of level dynamics of the adiabatic eigenstates of the autonomous system with $\alpha = \alpha_0$ independent of t. Under special initial conditions one may find quasi-elasticity as an intermediate case where the system evolves cyclically with $E(T) = E(0)$, T the period of the perturbation.

Acknowledgment

This work was supported in part by the University of Münster and by the National Natural Science Foundation, the Doctoral Education Fund of the State Education Commission and the Research Fund of Nuclear Industry of China.

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